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Why is the running vacuum energy more benign than the holographic Ricci dark energy?

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A very interesting generalization of the running vacuum energy density has been recently advanced [S. Basilakos, D. Polarski, and J. Sola, Phys. Rev. D **86**, 043010 (2012)]. The Friedmann equation of this model looks pretty much similar to that of a homogeneous and isotropic universe filled with an holographic Ricci dark energy (HRDE) component. Despite the analogy between these two models, it turns out that one of them, generalization of the running vacuum energy, is singularity-free in the future while the other, HRDE, is not. Indeed, a universe filled with an HRDE component can hit, for example, a big rip singularity. We clarify this issue by solving analytically the Friedmann equation for both models and analyzing the role played by the local conservation of the energy density of the different components when filling the universe. In addition, not everything is bad news about the HRDE. In fact, we point out that in some particular cases the HRDE, when endowed with a *negative* cosmological constant and in the absence of an explicit dark matter component, can mimic dark matter and explain the late-time cosmic acceleration of the universe through an asymptotically de Sitter universe.

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I. INTRODUCTION

From several astrophysical observations (supernovae type Ia [1], cosmic microwave background [2], large scale structure [3], etc.), it is now widely accepted that the Universe is undergoing a state of accelerating expansion. On the other hand, dark energy (characterized with negative pressure) is the simplest and may be the most physical cause for the current acceleration of the Universe [4].

Now what is dark energy? We are far from giving an answer to this question. Nevertheless, we would like to highlight that there are several promising candidates as theoretical directions to the dark energy problem from the point of view of fundamental physics. An important candidate, inspired on applying the holographic principle to the Universe as a whole, was advanced and named the holographic dark energy scenario [5,6] whose energy density is inversely proportional to the square of an appropriate length L that characterizes the size of the system, in this case the Universe, and representing the infrared (IR) cutoff of it. One of the natural choices of this length L is the inverse of the Hubble rate. However, this choice does not induce acceleration in a homogeneous and isotropic universe [6] (see Refs. [7,8] for an example where a modification of the model presented in Ref. [6] can explain the current acceleration of the Universe). Another choice for the length L was suggested by Gao *et al.* [9] (see also Ref. [10]), in which the IR cutoff of the holographic Ricci dark energy (HRDE) was taken to be the Ricci scalar curvature, i.e., $L^2 \propto 1/R$.

Another very interesting approach to explain the late-time speedup of the Universe is to invoke an evolving vacuum energy which was named the running vacuum energy [11–13]. This approach is related to the cosmological constant Λ (problem), where Λ would be no longer constant along the expansion of the Universe but changing as predicted from the renormalization group equation for the vacuum energy [11]. This model has been recently generalized in Ref. [14], being almost undistinguishable from the standard ΛCDM and baptized the generalized running vacuum energy (GRVE). Phenomenologically, this model has a formal analogy to the HRDE, where a combination of $\dot{H}$ and H^2 is present on its energy density [cf. Eq. (2.3)] with $\dot{H}$ the cosmic time derivative of the Hubble rate. Despite this analogy, the GRVE contains an additional constant term in order to allow for a transition from a decelerated to an accelerated expansion [14].

In addition, the issue of the late-time singularities has been considered in the context of dark energy cosmology (cf., for example, Refs. [15–18]). The main purpose of this work is to point out the nature of the possible future singularities in the framework of the two recently proposed models of dark energy cosmology: the HRDE and the GRVE.

The paper is organized as follows. In Sec. II, we review the GRVE model within the context of a Friedmann-Lemaître-Robertson-Walker (FLRW) background. We will then study the late-time behavior of this cosmological model. We will further analyze in Sec. III the late-time behavior of a homogeneous and isotropic universe where the HRDE plays the role of dark energy. We point out also that in some particular cases the HRDE model, when endowed with a *negative* cosmological constant,

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can mimic dark matter and explain the late-time cosmic acceleration through an asymptotically de Sitter universe. Finally, in Sec. IV we will present our conclusions and outlook.

II. THE GENERALIZED RUNNING VACUUM ENERGY MODEL

We assume a spatially flat FLRW universe filled with matter, with energy density ρ and pressure p, and the GRVE playing the role of dark energy. Then, the evolution of the universe is described by [14]

$$\frac{\dot{a}^2}{a^2} = \frac{1}{3M_{\rm P}^2}(\rho + \rho_\Lambda(H, \dot{H})), \tag{2.1}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm P}^2}((\rho + 3p) + 2p_\Lambda(H, \dot{H})), \tag{2.2}$$

where $H = \dot{a}/a$ and $M_{\rm P}$ is the reduced Planck mass given as $M_{\rm P}^{-2} = 8\pi G$, G being the gravitational constant. Moreover, $\rho_\Lambda(H, \dot{H})$ is "the generalized running vacuum energy" density with $p_\Lambda(H, \dot{H})$ being the corresponding pressure [14]:

$$\rho_\Lambda(H, \dot{H}) = -p_\Lambda(H, \dot{H}) = 3M_{\rm P}^2(C_0 + C_H H^2 + C_{\dot{H}}\dot{H}), \tag{2.3}$$

in which the equation of state satisfying $w_\Lambda = p_\Lambda/\rho_\Lambda = -1$ is assumed. The parameters C_0, C_H and $C_{\dot{H}}$ are constants. In addition, the constant[1] C_0 is given by [11,14]

$$C_0 = \frac{\rho_\Lambda^0}{3M_{\rm P}^2} - \nu H_0^2 - C_{\dot{H}}\dot{H}_0, \tag{2.4}$$

where

$$\nu = \sum_i \frac{B_i}{48\pi^2}\frac{M_i^2}{M_{\rm P}^2}, \tag{2.5}$$

and $\rho_\Lambda^0 = \rho_\Lambda(H_0)$ is the energy density defined at the present time t_0 or equivalently at the current Hubble rate $H_0 = H(t_0)$. In addition, M_i is the masses of particles contributing in the loops [12]. The dimensionless parameter ν provides the main coefficient of the β function for the running of the vacuum energy, and B_i are coefficients computed from the quantum loop contributions of fields with masses M_i [12]. Meanwhile, C_H and $C_{\dot{H}}$ are dimensionless coefficients that can be fitted to the observations. For convenience we will set henceforth, a new notation as $C_H \equiv \nu$ and $C_{\dot{H}} \equiv \frac{2}{3}\alpha$ (see also Footnote [1]), where α and ν are expected to be small (cf. Ref. [14]). On the other hand, ρ is the remaining matter energy density, given in the standard cosmological case as

$$\rho = \rho_{\rm m} + \rho_{\rm r}, \tag{2.6}$$

with $\rho_{\rm m}$ being the energy density of the nonrelativistic dustlike matter ($p_{\rm m} = w_{\rm m}\rho_{\rm m} = 0$) and $\rho_{\rm r}$ the energy density for radiation ($p_{\rm r} = w_{\rm r}\rho_{\rm r} = \frac{1}{3}\rho_{\rm r}$).

A characteristic of the GRVE model is that while the total energy density is conserved, a given energy density is not conserved. Therefore a local conservation law is employed on the GRVE setup, whereas it does not yield a conservation equation for each component separately. Following the approach used in Ref. [14] and by considering a decoupled conservation equation,[2] the solution for the matter components can be written as [14]

$$\rho_{\rm m} = \rho_{\rm m}^0 a^{-3\xi_{\rm m}}, \quad \text{where } \xi_{\rm m} \equiv \frac{1-\nu}{1-\alpha}, \tag{2.7}$$

$$\rho_{\rm r} = \rho_{\rm r}^0 a^{-4\xi_{\rm r}}, \quad \text{where } \xi_{\rm r} \equiv \frac{1-\nu}{1-4\alpha/3}, \tag{2.8}$$

where $\rho_{\rm m}^0$ and $\rho_{\rm r}^0$ are, respectively, the energy density of dust and radiation at the present time. Substituting now Eqs. (2.7), (2.8), and (2.3) in Eq. (2.1) we can rewrite the generalized Friedmann equation in the following form:

$$E^2 = \Omega_{\rm m}(1+z)^{3\xi_{\rm m}} + \Omega_{\rm r}(1+z)^{4\xi_{\rm r}} + \Omega_0 + \nu E^2 + \frac{2\alpha}{3H_0}\dot{E}, \tag{2.9}$$

where $E(z) = H/H_0$, z is the redshift and

$$\Omega_{\rm m} = \frac{\rho_{\rm m}^0}{3M_{\rm P}^2 H_0^2}, \qquad \Omega_{\rm r} = \frac{\rho_{\rm r}^0}{3M_{\rm P}^2 H_0^2}, \qquad \Omega_0 = \frac{C_0}{H_0^2}. \tag{2.10}$$

The dimensionless parameter for the generalized running vacuum energy density can be written as

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3M_{\rm P}^2 H_0^2} = \Omega_0 + \nu E^2 + \frac{2\alpha}{3H_0}\dot{E}. \tag{2.11}$$

Evaluating the Friedmann equation (2.9) at the present time, $z = 0$, gives a constraint on the cosmological parameters of the model which reads

$$1 = \Omega_{\rm m} + \Omega_{\rm r} + \Omega_\Lambda^0, \tag{2.12}$$

where Ω_Λ^0 is defined as

[1] The coefficient C_0 can be estimated by evaluating Eq. (2.3) at present. In addition, we assume that C_H can be evaluated as in the standard running vacuum energy, given by $C_{\rm H} \equiv \nu$, following the approach used in Ref. [14].

[2] The decoupling constant must vanish during the matter-dominated and radiation-dominated periods where matter and radiation cannot deviate from the standard scaling with the scale factor. The authors of Ref. [14] argued that this is a plausible condition that can be extended to the whole evolution of the universe.

$$\Omega_\Lambda^0 := \Omega_\Lambda(z=0) = \Omega_0 + \nu + \frac{2\alpha}{3H_0}\dot{E}(z=0). \quad (2.13)$$

The quantities $\dot{H}$ and H^2 are related through $\dot{H} = -(1+q)H^2$, where q is the deceleration parameter. Therefore, using Eq. (2.13), we can write down the deceleration parameter at the present time as

$$q_0 = -1 + \frac{3}{2\alpha}(\Omega_0 + \nu - \Omega_\Lambda^0). \quad (2.14)$$

Since the universe is currently accelerating, i.e., $q_0 < 0$, we obtain the constraint

$$\frac{3}{2\alpha}(\Omega_0 + \nu - \Omega_\Lambda^0) < 1. \quad (2.15)$$

Notice that a successful cosmological model must be able to produce an accelerated expansion at very low redshifts.

It is convenient to rewrite the generalized Friedmann equation (2.9) by introducing a new variable $x = -\ln(z+1) = \ln(a)$, as follows:

$$\frac{\dot{E}}{H_0} = -\frac{3}{2\alpha}[\Omega_m e^{-3\xi_m x} + \Omega_r e^{-4\xi_r x} + (\nu - 1)E^2 + \Omega_0]. \quad (2.16)$$

Substituting $\dot{E} = HdE/dx$ in Eq. (2.16) we can further rewrite the Friedmann equation in the following form:

$$\frac{dE^2}{dx} = -\frac{3}{\alpha}[\Omega_m e^{-3\xi_m x} + \Omega_r e^{-4\xi_r x} + (\nu - 1)E^2 + \Omega_0]. \quad (2.17)$$

Solving Eq. (2.17) and rewriting the result in terms of the redshift, we obtain

$$E^2(z) = E_m(1+z)^{3\xi_m} + E_r(1+z)^{4\xi_r} + E_0 + E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)}, \quad (2.18)$$

where

$$E_0 = \frac{\Omega_0}{(1-\nu)}, \qquad E_m = \frac{\Omega_m}{\xi_m}, \qquad E_r = \frac{\Omega_r}{\xi_r}, \quad (2.19)$$

and E_ν is an integration constant. Those constants are constrained by

$$1 = E_m + E_r + E_0 + E_\nu. \quad (2.20)$$

The first two terms in Eq. (2.18) are related to the energy densities of matter, ρ_m, and radiation, ρ_r, and the last two terms the GRVE density ρ_Λ. In the far future, as z decreases, the matter content of the universe is negligible, and therefore the energy density of the universe will be dominated by the GRVE density. By substituting Eq. (2.18) in the generalized Friedmann equation (2.16), we get the first time derivative for the Hubble parameter:

$$\frac{\dot{E}}{H_0} = -\frac{1}{2}\left[3\Omega_m(1+z)^{3\xi_m} + 4\Omega_r(1+z)^{4\xi_r} + \frac{3}{\alpha}(\nu-1)E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)}\right]. \quad (2.21)$$

Furthermore, using the solution (2.18) and its time derivative (2.21) in Eq. (2.3), we obtain the GRVE density defined as follows:

$$\rho_\Lambda = 3H_0^2 M_P^2\left[(\nu - \alpha\xi_m)E_m(1+z)^{3\xi_m} + \left(\nu - \frac{4}{3}\alpha\xi_r\right)E_r(1+z)^{4\xi_r} + E_0 + E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)}\right]. \quad (2.22)$$

It can be seen that at present $\rho_\Lambda = M_P^2\Lambda$ reduces to an effective cosmological constant given by

$$\Lambda = 3H_0^2\left[(\nu - \alpha\xi_m)E_m + \left(\nu - \frac{4}{3}\alpha\xi_r\right)E_r + E_0 + E_\nu\right]. \quad (2.23)$$

Using Eq. (2.20), we can rewrite E_ν in terms of E_m, E_r and E_0. Finally, substituting this expression of E_ν in Eq. (2.23) the constraint (2.12) is recovered, where $\Lambda/3H_0^2 \equiv \Omega_\Lambda^0$.

The GRVE density (2.22) is obtained herein as a function of the energy density of the matter component and the radiation one, plus the last two terms which play an important role on the fate of the universe at the very low redshift regime in the future. More precisely, in the presence of the last term in Eq. (2.22), if[3] $\frac{3}{\alpha}(\nu - 1) < 0$, then the Hubble rate and its time derivative diverge and, hence, the universe undergoes a big rip singularity at $z = -1$ [15].

On the other hand, the total energy density of the universe, $\rho_{tot} = \rho_m + \rho_r + \rho_\Lambda$, reads

$$\rho_{tot} = 3H_0^2 M_P^2[E_m(1+z)^{3\xi_m} + E_r(1+z)^{4\xi_r} + E_0 + E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)}]. \quad (2.24)$$

The total energy density (2.24) must satisfy the conservation law

$$\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0, \quad (2.25)$$

with p_{tot} being the total pressure of the cosmological system: $p_{tot} = p_m + p_r + p_\Lambda$, where $p_m = 0$, $p_r = \frac{1}{3}\rho_r$, and $p_\Lambda = -\rho_\Lambda$. Therefore, the total conservation law (2.25) reads

$$(\Omega_m - \xi_m E_m)(1+z)^{3\xi_m} + \frac{4}{3}(\Omega_r - \xi_r E_r)(1+z)^{4\xi_r} + \left(\frac{1-\nu}{\alpha}\right)E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)} = 0. \quad (2.26)$$

[3]From now on, we will focus on the case $\frac{3}{\alpha}(\nu - 1) < 0$, because we are trying to see if this model can avoid the big rip singularity which appears precisely in this case.

By using the definitions (2.19) in Eq. (2.26), the first two terms vanish spontaneously, but the last term does not. Thus, the conservation equation implies

$$\left(\frac{1-\nu}{\alpha}\right)E_\nu(1+z)^{\frac{3}{\alpha}(\nu-1)}=0. \tag{2.27}$$

The conservation equation (2.25) [or equivalently Eq. (2.26)] is fulfilled only when $\nu=1$ or $E_\nu=0$ [cf. Eq. (2.27)]. In the former case ($\nu=1$), the last term in equality (2.22) behaves as a cosmological constant. In the latter case ($E_\nu=0$), the last term in Eq. (2.22) vanishes which corresponds exactly to the case analyzed in Ref. [14]. Indeed, the local conservation law constrains the GRVE density leading to the evolution of ρ_Λ *only* in terms of the matter energy components and an effective cosmological constant. Therefore, since the energy density of the matter and radiation vanish in the far future ($z\to-1$), then the GRVE density remains finite. Therefore, the GRVE scenario is free of future singularities and becomes asymptotically de Sitter.

III. THE HOLOGRAPHIC RICCI DARK ENERGY MODEL

In this section, we consider a flat FLRW universe in the presence of nonrelativistic matter, radiation and an HRDE component [9]. The Friedmann equation for this model reads

$$\frac{\dot a^2}{a^2}=\frac{1}{3M_{\rm P}^2}(\rho_{\rm m}+\rho_{\rm r}+\rho_{\rm H}), \tag{3.1}$$

where $\rho_{\rm m}$, $\rho_{\rm r}$ and $\rho_{\rm H}$ denote the energy density of matter, radiation and the HRDE component, respectively. The pressureless matter $\rho_{\rm m}$ and radiation $\rho_{\rm r}$ are self-conserved unlike in the GRVE model previously discussed, that is,

$$\rho_{\rm m}=3M_{\rm P}^2H_0^2\Omega_{\rm m}(1+z)^3,\qquad \rho_{\rm r}=3M_{\rm P}^2H_0^2\Omega_{\rm r}(1+z)^4, \tag{3.2}$$

where $\Omega_{\rm m}$ and $\Omega_{\rm r}$ are the dimensionless energy density parameters defined in Eq. (2.10). Furthermore, the HRDE density is proportional to the inverse of the Ricci scalar curvature radius $\mathcal{R}$:

$$\mathcal{R}=6(\dot H+2H^2). \tag{3.3}$$

So, the HRDE density is defined as [9]

$$\rho_{\rm H}=3\beta M_{\rm P}^2\left(\frac{1}{2}\frac{dH^2}{dx}+2H^2\right), \tag{3.4}$$

where $\beta=c^2$ is a dimensionless parameter that measures the strength of the holographic component. By rewriting Eq. (3.2) in terms of $x=-\ln(z+1)$ and substituting it together with Eq. (3.4) in Eq. (3.1) the Friedmann equation can be rewritten as [9]

$$E^2=\Omega_{\rm m}e^{-3x}+\Omega_{\rm r}e^{-4x}+\beta\left(\frac{1}{2}\frac{dE^2}{dx}+2E^2\right). \tag{3.5}$$

Therefore, the dimensionless energy density parameter of the HRDE component can be written as

$$\Omega_{\rm H}=\beta\left(\frac{1}{2}\frac{dE^2}{dx}+2E^2\right). \tag{3.6}$$

Notice that the Friedmann equation (3.5) is pretty much similar to the Friedmann equation (2.9) for the GRVE model. There is a difference which is based on the fact that Eq. (2.9) contains a phenomenological cosmological constant which is absent in Eq. (3.5). For the sake of completeness, we will consider as well a phenomenological cosmological constant $\tilde\Omega_0$, in the model discussed on the present section; therefore, Eq. (3.5) will be rewritten as

$$E^2=\Omega_{\rm m}e^{-3x}+\Omega_{\rm r}e^{-4x}+\tilde\Omega_0+\beta\left(\frac{1}{2}\frac{dE^2}{dx}+2E^2\right), \tag{3.7}$$

and $\tilde\rho_0\equiv 3M_{\rm P}^2H_0^2\tilde\Omega_0$ is a constant. We will compare the model resulting from Eq. (3.7) with the one of the previous section in the presence or absence of the cosmological constant $\tilde\Omega_0$.

By evaluating the Friedmann equation (3.7) at the present time, we obtain a constraint on the dimensionless parameters of the model:

$$1=\Omega_{\rm m}+\Omega_{\rm r}+\tilde\Omega_0+\Omega_{\rm H_0}. \tag{3.8}$$

After solving the Friedmann equation (3.5), we get

$$E^2(z)=\frac{2}{2-\beta}\Omega_{\rm m}(1+z)^3+\Omega_{\rm r}(1+z)^4+\Omega_\beta(1+z)^{4-\frac{2}{\beta}}+\frac{\tilde\Omega_0}{1-2\beta}, \tag{3.9}$$

where $\beta\neq\frac{1}{2}$, 2 and Ω_β is an integration constant. Then, by evaluating the solution (3.9) at the present time, we obtain

$$1=\frac{2\Omega_{\rm m}}{2-\beta}+\Omega_{\rm r}+\Omega_\beta+\frac{\tilde\Omega_0}{1-2\beta}, \tag{3.10}$$

which is a complementary constraint to that given in Eq. (3.8).

Substituting $E(z)$ given in Eq. (3.9) in Eq. (3.4), we obtain the HRDE density:

$$\rho_{\rm H}=3M_{\rm P}^2H_0^2\left[\frac{\beta}{2-\beta}\Omega_{\rm m}(1+z)^3+\Omega_\beta(1+z)^{4-\frac{2}{\beta}}+\frac{2\beta}{1-2\beta}\tilde\Omega_0\right]. \tag{3.11}$$

Notice that, in the HRDE model, it is assumed that the energy density of the different components filling the universe is conserved and in particular the one corresponding to the HRDE. So, by substituting the energy density (3.11) in the conservation law $\dot{\rho}_{\rm H} + 3H(\rho_{\rm H} + p_{\rm H}) = 0$, we obtain the HRDE pressure $p_{\rm H}$:

$$p_{\rm H} = -3M_{\rm P}^2 H_0^2 \left[\frac{2\beta}{1-2\beta}\tilde{\Omega}_0 + \left(\frac{2}{3\beta} - \frac{1}{3}\right)\Omega_\beta (1+z)^{4-\frac{2}{\beta}}\right]. \tag{3.12}$$

Finally, the total energy density reads

$$\rho_{\rm tot} = 3M_{\rm P}^2 H_0^2 \left[\frac{2\Omega_{\rm m}}{2-\beta}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + \Omega_\beta (1+z)^{4-\frac{2}{\beta}} + \frac{\tilde{\Omega}_0}{1-2\beta}\right]. \tag{3.13}$$

Before continuing, we notice that the term $(1+z)^{4-\frac{2}{\beta}}$ on the previous equation induces acceleration if and only if $0 < \beta < 1$. We will impose this condition: $0 < \beta < 1$, to ensure late-time acceleration even in the absence of a cosmological constant $\tilde{\Omega}_0$. In the far future, as z tends to -1, the universe would be dominated by the holographic dark energy or the cosmological constant $\tilde{\Omega}_0$. For a positive cosmological constant ($\tilde{\Omega}_0 > 0$), if the range of the holographic parameter satisfies $0 < \beta < \frac{1}{2}$, then the energy density (3.13) and the Hubble rate (3.9) diverge as well as $\dot{H}$ and p_H; therefore, the universe hits a big rip singularity [15,16]. Notice that for $0 < \beta < \frac{1}{2}$, the future singularity is avoided only for vanishing Ω_β. However, Ω_β has a crucial role in the acceleration of an "holographic" universe; hence, the big rip singularity is unavoidable within the HRDE scenario unless $1 > \beta > \frac{1}{2}$. In addition, for the range of the HRDE parameter $1 > \beta > \frac{1}{2}$, the last term in Eq. (3.9) becomes negative, whereas the rest of the terms are positive. Therefore, as the universe evolves, at some redshift $z_{\rm b}$ in the future, the positive and negative terms in Eq. (3.9) will be canceled, and hence the Hubble parameter vanishes at that redshift. Using the relation $\dot{E}/H_0 = dE^2/(2dx)$ and Eq. (3.9), we get the time derivative of the Hubble rate:

$$\frac{\dot{E}}{H_0} = -\frac{3}{2-\beta}\Omega_{\rm m}(1+z)^3 - 2\Omega_{\rm r}(1+z)^4 - \left(\frac{1}{\beta} - 2\right)\Omega_\beta (1+z)^{4-\frac{2}{\beta}}, \tag{3.14}$$

which remains finite at $z_{\rm b}$ where the Hubble rate vanishes; i.e., $\dot{E}(z_{\rm b}) = {\rm const}$ when $E(z_{\rm b}) = 0$. Therefore, the universe in this case will bounce in the future and contract afterwards.

For a negative cosmological constant ($\tilde{\Omega}_0 < 0$), a similar analysis shows that, if $0 < \beta < \frac{1}{2}$, at some redshift z_b the universe will bounce in the future. If $\frac{1}{2} < \beta < 1$, the universe is asymptotically de Sitter.

In order to complete the discussion of this section, we will analyze the cases of the holographic parameter β, when $\beta = \frac{1}{2}$ and $\beta = 2$.

For the case of $\beta = \frac{1}{2}$, the solution for the Friedmann equation (3.7) in terms of the redshift reads

$$E^2(z) = \frac{4}{3}\Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + 4\tilde{\Omega}_0 \ln(1+z), \tag{3.15}$$

where the time derivative of the Hubble parameter is given by

$$\frac{\dot{E}}{H_0} = -2[\Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + \tilde{\Omega}_0]. \tag{3.16}$$

The constraint (3.8) at the present time, in this case, can be written as

$$1 = \frac{4}{3}\Omega_{\rm m} + \Omega_{\rm r}. \tag{3.17}$$

Moreover, the holographic energy density is given by

$$\rho_{\rm H} = 3M_{\rm P}^2 H_0^2 \left[\frac{\Omega_{\rm m}}{3}(1+z)^3 + 4\tilde{\Omega}_0 \ln(1+z) - \tilde{\Omega}_0\right], \tag{3.18}$$

with the holographic pressure reading

$$p_{\rm H} = 3M_{\rm P}^2 H_0^2 \tilde{\Omega}_0 \left[\frac{7}{3} - 4\ln(1+z)\right]. \tag{3.19}$$

Then, using Eqs. (3.1) and (3.15), the total energy density filling the universe would be

$$\rho_{\rm tot} = 3M_{\rm P}^2 H_0^2 \left[\frac{4}{3}\Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4 + 4\tilde{\Omega}_0 \ln(1+z)\right]. \tag{3.20}$$

On the one hand, for a positive cosmological constant ($\tilde{\Omega}_0 > 0$) the first two terms in Eq. (3.15) are positive, whereas the last term is negative as the universe evolves at late time. Therefore, there exists a moment in the future, namely, at a redshift $z_{\rm b}$, at which the Hubble rate vanishes, whereas the time derivative of the Hubble rate remains finite. Therefore, the universe bounces in the future.

On the other hand, for a negative cosmological constant ($\tilde{\Omega}_0 < 0$), all terms in Eq. (3.15) are positive in the future. In the far future, the Hubble rate diverges at $z = -1$ while its cosmic time derivative is finite. It can be checked that this event happens at an infinite cosmic time. Therefore, the universe undergoes a kind of smooth little rip singularity in the far future [17,18]. This kind of event was named by one of us "the little sibling of the big rip singularity" [19].

Finally, in the absence of the cosmological constant ($\tilde{\Omega}_0 = 0$), the Hubble rate and its time derivative vanish at $z = -1$; the universe becomes Minkowskian in the far future.

In summary, it is surprising that, in a HRDE model with $\beta = 1/2$, the presence of a positive cosmological constant can induce a bounce while the presence of a negative cosmological constant can induce a little sibling of the big rip singularity [19] (a smoother version of the little rip [17,18]).

If the holographic parameter is such that $\beta = 2$, the solution of the Friedmann equation (3.7) reads

$$E^2(z) = \Omega_{\rm m}(1+z)^3\ln(1+z) + \Omega_{\rm r}(1+z)^4 + \Omega_2(1+z)^3 - \frac{\tilde{\Omega}_0}{3}, \tag{3.21}$$

where Ω_2 is a constant. Furthermore, the constraint equation (3.8) for this solution reads

$$1 = \Omega_{\rm r} + \Omega_2 - \frac{\tilde{\Omega}_0}{3}. \tag{3.22}$$

The time derivative of the Hubble rate (3.21) is obtained easily as follows:

$$\frac{\dot{E}}{H_0} = -\frac{1}{2}[\Omega_{\rm m}(1+z)^3 + 3\Omega_2(1+z)^3 + 4\Omega_{\rm r}(1+z)^4 + \Omega_{\rm m}(1+z)^3\ln(1+z)]. \tag{3.23}$$

Substituting Eq. (3.21) in Eq. (3.4), the holographic energy density $\rho_{\rm H}$ reads

$$\rho_{\rm H} = 3M_{\rm P}^2H_0^2\bigg[\Omega_2(1+z)^3 - \Omega_{\rm m}(1+z)^3 \times \Omega_{\rm m}(1+z)^3\ln(1+z) - \frac{4}{3}\tilde{\Omega}_0\bigg]. \tag{3.24}$$

Finally, the holographic pressure $p_{\rm H}$ can be obtained by substituting Eq. (3.24) in the conservation equation:

$$p_{\rm H} = M_{\rm P}^2H_0^2[\Omega_{\rm m}(1+z)^3 + 4\tilde{\Omega}_0]. \tag{3.25}$$

Furthermore, the total energy density of the universe when the holographic parameter fulfils $\beta = 2$ reads

$$\rho_{\rm tot} = 3M_{\rm P}^2H_0^2\bigg[\Omega_{\rm m}(1+z)^3\ln(1+z) + \Omega_{\rm r}(1+z)^4 + \Omega_2(1+z)^3 - \frac{\tilde{\Omega}_0}{3}\bigg]. \tag{3.26}$$

This solution shows that, for a given cosmological constant no matter its sign (i.e., $\tilde{\Omega}_0 > 0$, $\tilde{\Omega}_0 < 0$, and $\tilde{\Omega}_0 = 0$), the Hubble rate (3.21) vanishes at some redshift $z_{\rm b}$ in the future, whereas the time derivative of Hubble rate (3.23) remains finite; therefore, the universe hits a bounce at $z_{\rm b}$ in the future.

On the other hand, if $\Omega_{\rm m} = 0$, the Hubble rate (3.21) reduces to

$$E^2(z) = \Omega_{\rm r}(1+z)^4 + \Omega_2(1+z)^3 - \frac{\tilde{\Omega}_0}{3}. \tag{3.27}$$

Notice that the evolution of the term proportional to Ω_2 in Eq. (3.27) is dustlike, indicating that the HRDE can play the role of dark matter in this case. In addition, for a positive cosmological constant, the Hubble rate would vanish at some $z_b(\neq -1)$ in the future. Furthermore, the time derivative of the Hubble rate defined as

$$\frac{\dot{E}}{H_0} = -\frac{1}{2}[3\Omega_2(1+z)^3 + 4\Omega_{\rm r}(1+z)^4] \tag{3.28}$$

remains finite at late time; therefore, the universe hits a bounce within a finite time in the future. In this case there will be no acceleration in the far future of the universe. On the other hand, for a negative cosmological constant, the Hubble rate (3.27) remains finite and nonzero at $z = -1$ while its time derivative (3.28) vanishes at $z = -1$. Therefore, the universe becomes de Sitter in the far future. Consequently, the presence of a negative cosmological constant in this case can explain the late-time acceleration of the universe.

IV. CONCLUSION

We have analyzed in this paper the issues of future singularities in the context of two recently proposed models for dark energy. We considered the GRVE (cf. Sec. II and Ref. [14]) and the HRDE scenario (cf. Sec. III and Refs. [9,18]), separately, within the context of FLRW cosmology. Even though the Friedmann equation of both models looks pretty much similar [cf. Eqs. (2.9) and (3.5)], there is a difference which is based on the fact that Eq. (2.9) contains a phenomenological cosmological constant which is absent in Eq. (3.5). For the sake of completeness and in order to compare both models, we have considered as well a phenomenological cosmological constant on the HRDE scenario.

On the one hand, in the GRVE model, a local conservation law constrains the total energy density of the universe but the energy momentum tensor of each different component filling the universe is not self-conserved. Indeed, this conservation law provides an energy transfer between the matter components and the running vacuum energy density leading to a running vacuum energy density dominating the universe at late time. It turns out that, in the far future, the universe is asymptotically de Sitter and free from singularities.

On the other hand, each component of the energy density in the HRDE model is self-conserved, leading to a different behavior of the HRDE energy density depending on the HRDE parameter β. In this case, the universe may hit a big rip [15], a smoother version of the little rip [17], named recently the little sibling of the big rip singularity, a bounce or it can even become asymptotically Minkowski or de Sitter. It is worthy to stress that this model becomes asymptotically de Sitter if and only if the HRDE is endowed with a *negative* cosmological constant. In addition, in the particular case $\beta = 2$, the HRDE can mimic dark matter if $\Omega_m = 0$.

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